Çankaya University
Department of Mathematics
2016 - 2017 Summer Semester

MATH 156 - Calculus for Engineering II

First Midterm Examination

1) Are the following sequences convergent or divergent? Explain.

(7 pts.) **a)** $a_n = e^{\sqrt{n}}$

(8 pts.) **b)** $a_n = \dfrac{\ln n}{n^2}$

2) Are the following series convergent or divergent? Explain.

(7 pts.) **a)** $\displaystyle\sum_{n=1}^{\infty} \cos\left(\frac{n}{1+n^2}\right)$

(8 pts.) **b)** $\displaystyle\sum_{n=1}^{\infty} \frac{2n}{e^{n^2}}$

3) Are the following series convergent or divergent? Explain.

(7 pts.) **a)** $\displaystyle\sum_{n=1}^{\infty} \frac{n\sqrt{n}+n+\ln n}{1+2n+3n^3}$

(8 pts.) **b)** $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^2\,5^n}{(2n)!}$

4) Are the following series absolutely convergent, conditionally convergent or divergent? Explain.

(10 pts.) **a)** $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n}{1+n^3}$

(10 pts.) **b)** $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n}{1+n^2}$

5) Find the radius and interval of convergence of the series $\displaystyle\sum_{n=1}^{\infty} \frac{(2x+5)^n}{3^n\sqrt{n}}$.

6) (7 pts.) **a)** Find the limit $\displaystyle\lim_{x\to 0} \frac{e^{x^2}-1-x^2}{2\cos x - 2 + x^2}$.

(6 pts.) **b)** Find the sum of the series $0.4 + 0.16 + 0.064 + 0.0256 + \cdots$

(7 pts.) **c)** Find the sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\,7^n}$

Answers

1) **a)** Limit is ∞, sequence is divergent.

b) Using L'Hôpital's Rule we find that limit is 0, sequence is convergent.

2) **a)** $\lim\limits_{n\to\infty} \cos\left(\dfrac{n}{1+n^2}\right) = \cos 0 = 1$, divergent by n^{th} term test.

b) $\displaystyle\int_1^\infty \frac{2x\,dx}{e^{x^2}} = \frac{1}{e}$, convergent by integral test.

3) **a)** Consider $\displaystyle\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$. It is convergent by $p-$test, and $\displaystyle\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{1}{3}$,

so $\displaystyle\sum_{n=1}^{\infty} a_n$ is convergent by limit comparison test.

b) $\displaystyle\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{5}{4}$, divergent by Ratio Test.

4) **a)** $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ is convergent by $p-$test and $\displaystyle\sum_{n=1}^{\infty} \frac{n}{1+n^3}$ is convergent by comparison test.

So $\displaystyle\sum_{n=1}^{\infty} a_n$ is absolutely convergent by Root Test.

b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n}{1+n^2}$ is convergent by alternating series test but $\displaystyle\sum_{n=1}^{\infty} \frac{n}{1+n^2}$ is divergent by limit comparison test (compare with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$) Therefore the given series is conditionally convergent.

5) Using root test we obtain $\left|x + \dfrac{5}{2}\right| < \dfrac{3}{2}$ in other words: $-4 < x < -1$

At $x = -1$ we obtain $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$, divergent by $p-$test.

At $x = -4$ we obtain $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$, convergent by alternating series test.

Interval of convergence: $[-4, -1)$.

6) **a)** 6

b) $0.4\left(1 + 0.4 + 0.4^2 + \cdots\right) = 0.4 \cdot \dfrac{1}{1-0.4} = \dfrac{2}{3}$

c) $\ln\left(\dfrac{7}{6}\right)$